Transformations of Exponential Functions

- To graph an exponential function of the form $y = a(c)^{b(x-h)} + k$, apply transformations to the base function, $y = c^x$, where c > 0. Each of the parameters, a, b, h, and k, is associated with a particular transformation.

Example 1: Translations of Exponential Functions

Consider the exponential function $y = 2^x$. For each of the transformed functions,

- State the parameter and describe the transformation.
- Graph the base function and the transformed function on the same grid.
- Describe any changes to the domain, range, intercepts, and equation of the horizontal asymptote.
- Explain the effect of the transformation on an arbitrary point, (x,y), on the graph of the base function.

a. $y = 2^x + 3$

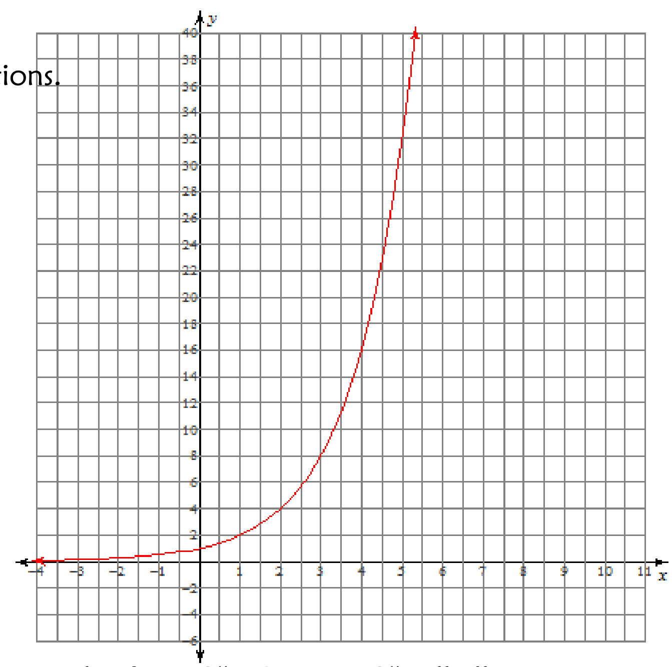
Solution:

a. $y = 2^x + 3$

Complete each table of values and then graph the functions.

$y = 2^x$

x	y
-2	
-1	
0	
1	
2	
3	
4	
5	



$y = 2^x + 3$

x	y
-2	
-1	
0	
1	
2	
3	
4	
5	

Comparing the graph of $y = 2^x + 3$ to $y = 2^x$ will allow you to determine the value of the parameter k. k = ___________

This transformation indicates that the graph of $y = 2^x + 3$ moves ________________________ compared to the graph of $y = 2^x$.

For the function $y = 2^x + 3$, state the

Domain: _____________________ Range: _____________________________

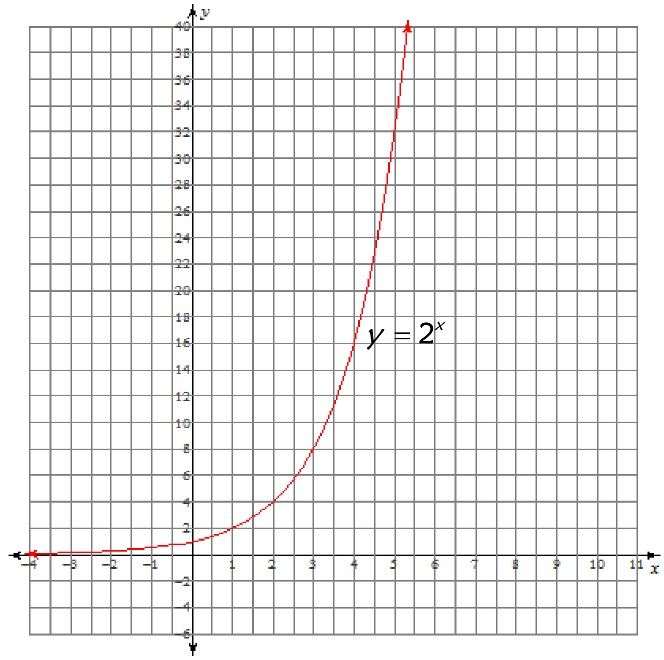
x-intercept: __________________ y-intercept: _______________________

Equation of the horizontal asymptote: ___________ Mapping rule: ________________________

b. $y = 2^{x-5}$

$y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16
5	32



Complete the table of values. Draw the graph of the transformed function.

$y = 2^{x-5}$ Mapping Rule: ____________________

X	y

Comparing the graph of $y = 2^{x-5}$ to $y = 2^x$ will allow you to determine the value of the parameter h. h= ________________.

This transformation indicates that the graph of $y = 2^{x-5}$ moves ____ compared to the graph of $y = 2^x$.

For the function $y = 2^{x-5}$, state the

Domain: ___________________________

Range: ____________________

x-intercept: ____________________

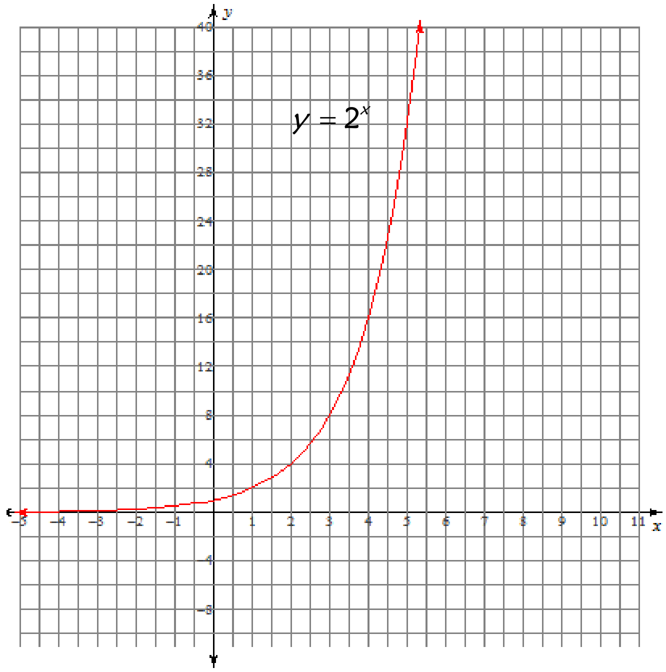
y-intercept: ___________

Equation of the horizontal asymptote: __________

c. $y+4=2^{x+1}$

$y=2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16
5	32



Complete the table of values. Draw the graph of the transformed function.

$y+4=2^{x+1}$ Mapping Rule: ___________________________

x	y

Comparing $y+4=2^{x+1}$ to $y=2^x$ will allow you to determine the value of the parameters h and k. h= ____________while k = __________.

This transformation indicates that the graph of $y+4=2^{x+1}$ moves

____

compared to the graph of $y=2^x$

For the function $y+4=2^{x+1}$, state the

Domain: ___________________________ Range: _____________________________

x-intercept: ______________ y-intercept: ___________

Equation of the horizontal asymptote: _________

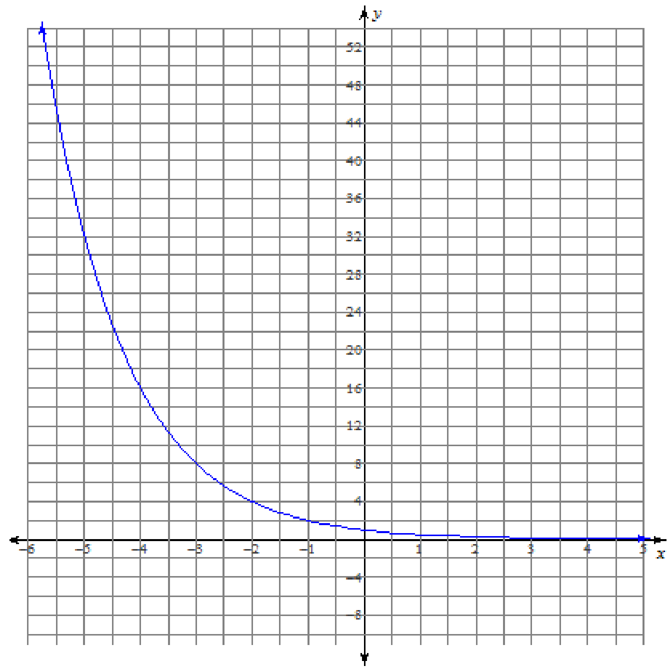
Example 2: Stretches of Exponential Functions

Consider the exponential function $y=\left(\frac{1}{2}\right)^x$. For each of the following transformations,

- State the parameter and describe the corresponding transformation
- Graph the base function and the transformed function on the same grid
- Describe any changes to the domain, range, intercepts, and equation of the horizontal asymptote
- Show what happens to an arbitrary point, (x,y), on the graph of the base function

a. $y=2\left(\frac{1}{2}\right)^x$

Complete the tables of values.
Draw the graph of the functions.



$y=\left(\frac{1}{2}\right)^x$

x	Y
-4	
-3	
-2	
-1	
0	
1	
2	

$y=2\left(\frac{1}{2}\right)^x$ Mapping Rule: ______________________________

x	Y
-4	
-3	
-2	
-1	
0	
1	
2	

Comparing $y=2\left(\frac{1}{2}\right)^x$ to $y=\left(\frac{1}{2}\right)^x$ will allow you to determine the value of the parameter a. a = ________________

This transformation indicates that the graph of $y=2\left(\frac{1}{2}\right)^x$ stretches ____ compared to the graph of $y=\left(\frac{1}{2}\right)^x$. For the function $y=2\left(\frac{1}{2}\right)^x$, state the

Domain :___________________________ Range :_____________________________

x-intercept: ______________ y-intercept: ___________

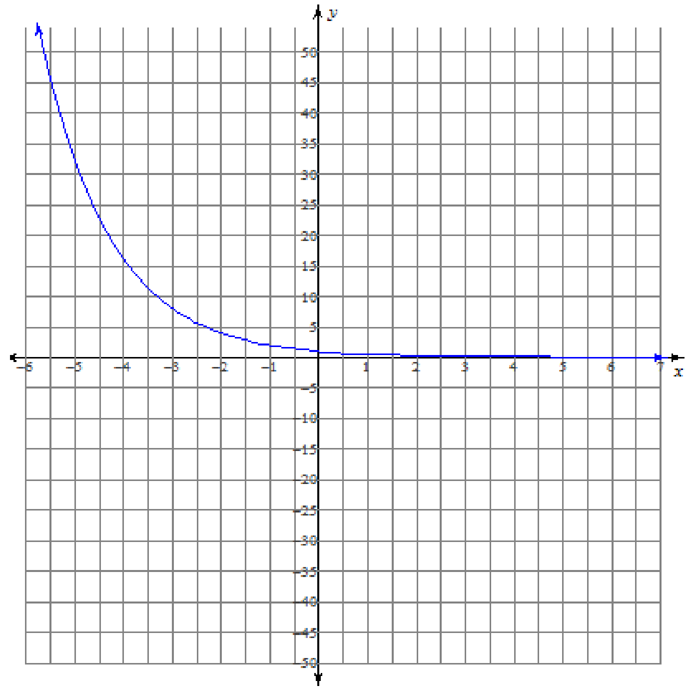
Equation of the horizontal asymptote; ________

b. $y = \left(\frac{1}{2}\right)^{2x}$

Complete the table of values.
Sketch the graphs of the functions.

$y = \left(\frac{1}{2}\right)^{x}$

x	y
-5	32
-4	16
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$



$y = \left(\frac{1}{2}\right)^{2x}$ Mapping Rule: ______________________________

x	y
	32
	16
	8
	4
	2
	1
	$\frac{1}{2}$
	$\frac{1}{4}$

Comparing $y = \left(\frac{1}{2}\right)^{2x}$ to $y = \left(\frac{1}{2}\right)^{x}$ will allow you to determine the value of the parameter b. b= ________________. This transformation indicates that the graph of $y = \left(\frac{1}{2}\right)^{2x}$ stretches ____ compared to the graph of $y = \left(\frac{1}{2}\right)^{x}$

For the function $y = \left(\frac{1}{2}\right)^{2x}$, state the

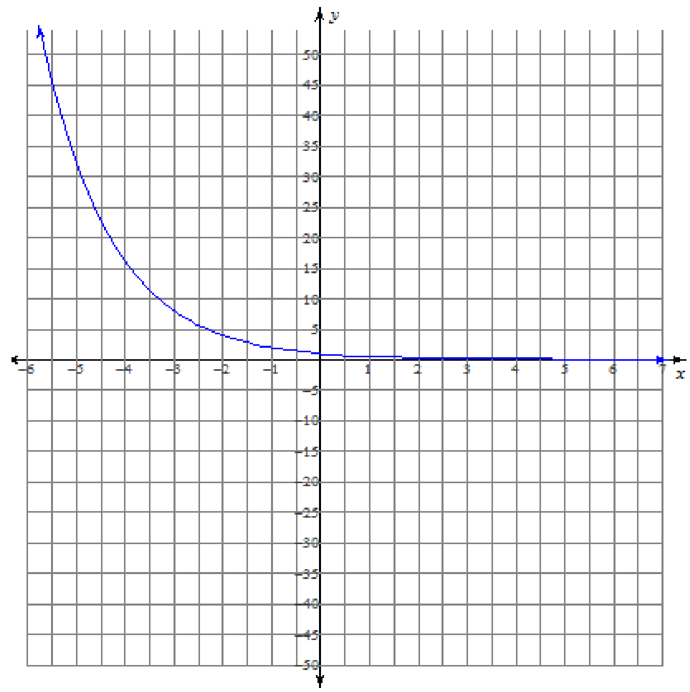
Domain: _________________________ Range :___________________________

x-intercept: _____________ y-intercept: __________

Equation of the horizontal asymptote; _______

c. $y = -3\left(\frac{1}{2}\right)^{-\frac{1}{2}x}$

Complete the table of values.
Sketch the graphs of the functions.



$y = \left(\frac{1}{2}\right)^{x}$

x	y
-4	16
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$

$y = -3\left(\frac{1}{2}\right)^{-\frac{1}{2}x}$ Mapping Rule: ______________________________

x	y

Comparing $y = -3\left(\frac{1}{2}\right)^{-\frac{1}{2}x}$ to $y = \left(\frac{1}{2}\right)^{x}$ will allow you to determine the value of the parameters a and b. a = _______ b = _________

This transformation indicates that the graph of $y = -3\left(\frac{1}{2}\right)^{-\frac{1}{2}x}$

____

________________________ as compared to the graph of $y = \left(\frac{1}{2}\right)^{x}$.

For the function $y = -3\left(\frac{1}{2}\right)^{-\frac{1}{2}x}$, state the

Domain: _______________________ Range: _________________________

x-intercept: ____________ y-intercept: _________

Equation of the horizontal asymptote: ______

Example 3: Combining Transformations of Exponential Functions

Consider the base function $y = 3^x$. For each transformed function,

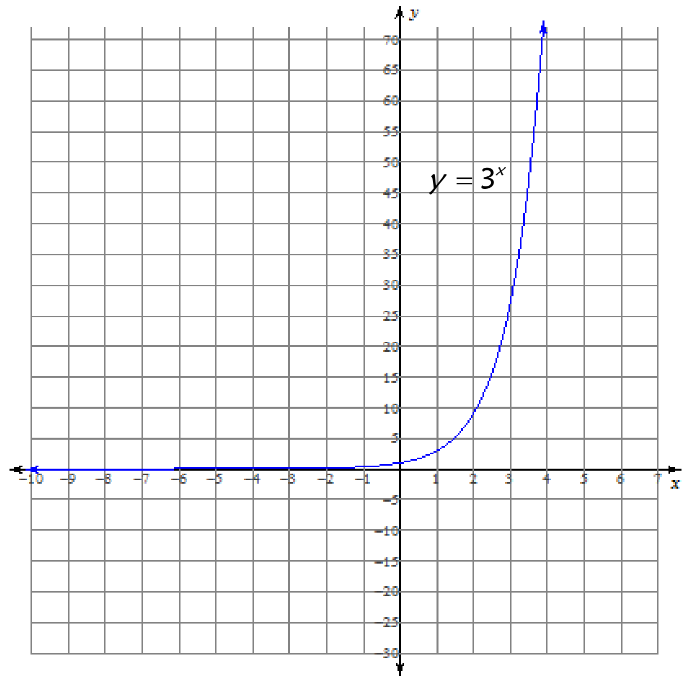
- State the parameters and describe the corresponding transformations
- Complete the table of values to show what happens to the given points under the transformations
- Sketch the graph of the base function and the transformed function.

a. Describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts

Complete each table of values and sketch the graph of the function $y = \frac{1}{3}(3)^{x+4}$.

$y = 3^x$

x	y
-2	
-1	
0	
1	
2	
3	
4	



$y = \frac{1}{3}(3)^{x+4}$ MAPPING RULE: ____________

X	y

Comparing $y = \frac{1}{3}(3)^{x+4}$ to $y = 3^x$ will allow you to determine the value of the parameters a, b, h and k.

a= _______ , b = _________, h=_________, k= ___________

These transformations indicate that the graph of $y = \frac{1}{3}(3)^{x+4}$

____

____

as compared to the graph of $y = 3^x$.

For the function $y = \frac{1}{3}(3)^{x+4}$, state the

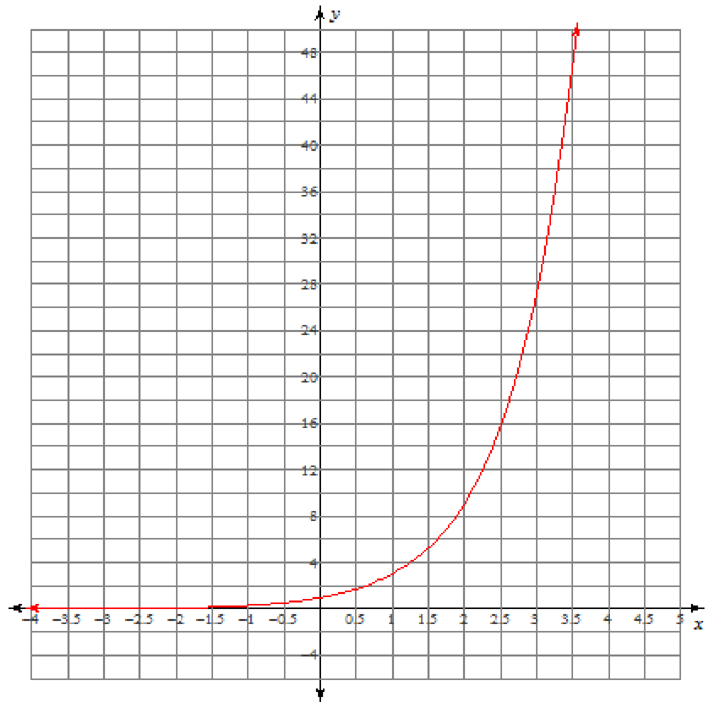
Domain ___________________________ Range _____________________________

x-intercept: ______________ y-intercept: ___________

Equation of Horizontal Asymptote : _______________

b. $y = 3^x$

X	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27



X	y

Complete the table of values and sketch the graph of $y = 2(3)^{-2(x-1)} - 5$

Mapping Rule: ______________________________

Comparing $y = 2(3)^{-2(x-1)} - 5$ to $y = 3^x$ will allow you to determine the value of the parameters a, b, h and k.

a= _______ , b = _________, h=_________, k= ___________

These transformations indicate that the graph of $y = 2(3)^{-2(x-1)} - 5$ is

____

____

as compared to the graph of $y = 3^x$.

For the function $y = 2(3)^{-2(x-1)} - 5$, state the

Domain :__________________________ Range: ____________________________

x-intercept: ______________ y-intercept: ___________

Equation of the horizontal asymptote: ___________ Mapping Rule $(x, y) \rightarrow$

Example 4A: Use Transformations of an Exponential Function to Model a Situation

The real estate board in a city announces that the current average price of a house in the city is $400 000. It predicts that average prices will double every 15 years.

a. Write a transformed exponential function in the form $y = a(c)^{b(x-h)} + k$ to model this situation. Justify your answer.

b. Describe how each of the parameters in the transformed function relates to the information provided.

c. Predict the value of the house after 10 years.

Solution:

a. Since the average price of a house doubles over a certain time interval, the base function is $P(t) = 2^t$, where P is the price of the house and t is the time.

The time is in intervals of 15 years, so t can be replaced by the rational exponent $\frac{r}{\ }$, where r represents the number of years. Therefore the function becomes P(r)=________________. The current price of a home is $400 000, so the P-intercept is (_________, __________). This means that there must be a vertical stretch by a factor of __________________. Therefore, the transformed function that models the price of the house is

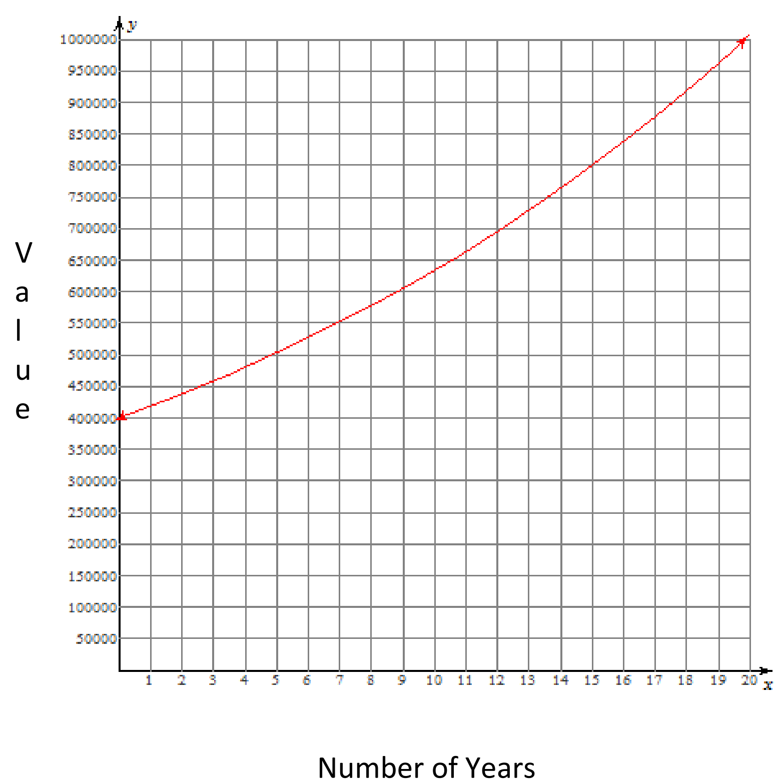
P(r)=______________________

b. Based on the function $y = a(c)^{b(x-h)} + k$, the parameters of the function are

b = , representing _______________

a = , representing _______________

c. Predict the value of the house after 10 years. Estimate by using the graph and then confirm with the equation.

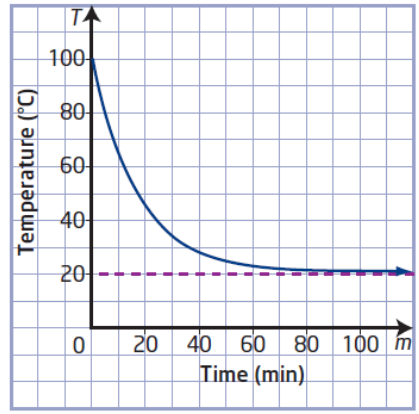


Example 4B: Use Transformations of an Exponential Function to Model a Situation

A cup of water is heated to 100 °C and then allowed to cool in a room with an air temperature of 20 °C. The temperature, T, in °C, is measured and plotted on a coordinate grid as a function of time, m, in minutes. It is found that the temperature of the water decreases exponentially at a rate of 25% every 5 minutes.

a. Write a transformed exponential function in the form $y = a(c)^{b(x-h)} + k$ to model this situation.

b. Use your equation to predict the temperature of the water after 1 hour.



Solution:

a. Determine the exponential function $y = a(c)^{b(x-h)} + k$:

- Since the water temperature decreases by 25% over a certain time interval, the base function is _________________, where T is the temperature of the water and t is the time, in 5-minute intervals.
- The time is in intervals of 5 minutes, so t can be replaced by the rational exponent __________, where m represents the number of minutes. Therefore, the function becomes ______________________.
- The horizontal asymptote is at _____________ (corresponding to the temperature of the room). This means that the function has been ____.
  This is represented in the function as ______________________________.
- The T-intercept of the graph is at _______________ (corresponding to the initial temperature of the water).
  This means that there must be a _______________________________. Use the coordinates of the T-intercept to determine the value of a:

$$T(m) = a(0.75)^{\frac{m}{5}} + 20$$

Therefore, the transformed function that models the temperature of the water is _________________________.

b. Temperature of the water after 1 hour:

$$T(m) = 80(0.75)^{\frac{m}{5}} + 20$$

$T(\_\_\_\_) =$

Note: From the graph, when m = 60, the value of T is approximately mple 5: