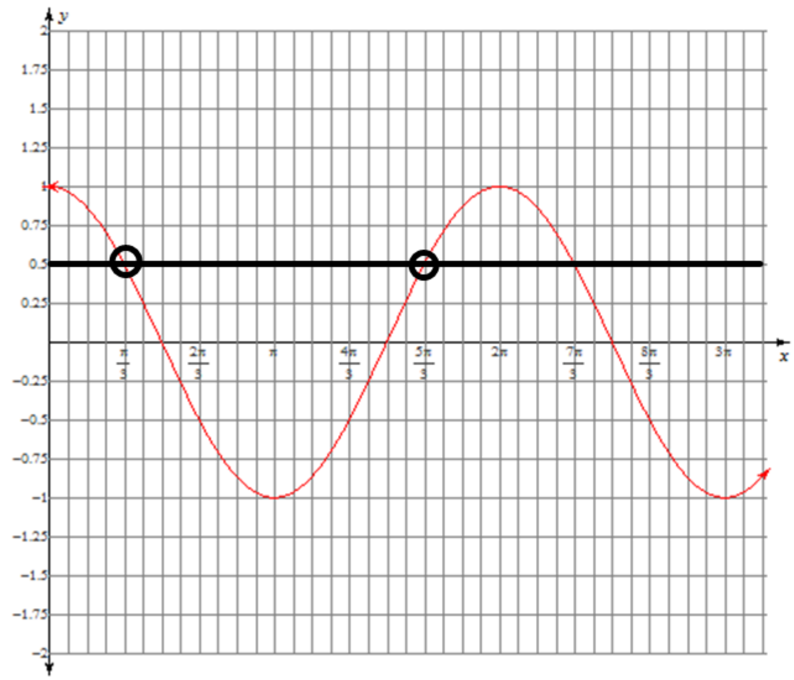
Equations and Graphs of Trigonometric Functions

When a question asks you to solve a system of trigonometric equations, you are looking for the values of θ that make both equations true. There are two ways to solve for θ: graphically and algebraically.

Example 1: Solving a Trigonometric Equation Graphically and Algebraically

Solve $\cos\theta = \frac{1}{2}$ in the domain $[0, 2\pi)$ and then state the general solutions.



GRAPHICALLY:

You are looking for the locations where the cosine curve intersects with the line $y = \frac{1}{2}$. Check for solutions in the domain $[0, 2\pi]$.

Reading the solutions off the graph:

$\frac{\pi}{3}$ AND $\frac{5\pi}{3}$

GENERAL SOLUTIONS

$\frac{\pi}{3} + 2\pi k, k \in I$ AND $\frac{5\pi}{3} + 2\pi k, k \in I$

ALGEBRAICALLY

$\cos\theta = \frac{1}{2}$

$\cos^{-1}(\cos\theta) = \cos^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{3}$ Quadrant I angle

$\theta = \frac{5\pi}{3}$ Quadrant IV angle

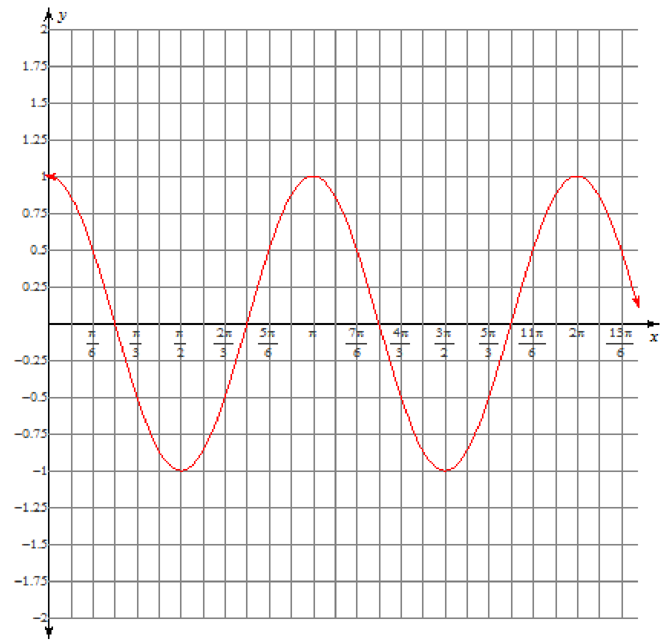
General Solutions: $\frac{\pi}{3} + 2\pi k, k \in I$ AND $\frac{5\pi}{3} + 2\pi k, k \in I$

Example: Solving a Trigonometric Equation Graphically and Algebraically

Solve $\cos 2\theta = \frac{1}{2}$ in the domain $[0, 2\pi)$ graphically and algebraically and then state the general solutions.

GRAPHICALLY

The points of intersection between the graphs of $y = \cos 2\theta$ and $y = \frac{1}{2}$ in the domain $[0, 2\pi)$ are:



GENERAL SOLUTIONS:

ALGEBRAICALLY:

Solve $\cos 2\theta = \frac{1}{2}$

Step 1: Let a $= 2\theta$

Step2 : $\cos a = \frac{1}{2}$

Step 3: General Solutions $a = \frac{\pi}{3} + 2\pi k, k \in I$ AND $a = \frac{5\pi}{3} + 2\pi k, k \in I$

However we were asked to solve for θ not a. So to continue

Step4 : $2\theta = \frac{\pi}{3} + 2\pi k, k \in I$ AND $2\theta = \frac{5\pi}{3} + 2\pi k, k \in I$

Step5: $\theta = \frac{\pi}{6} \pm \pi k, k \in I$ *AND* $\theta = \frac{5\pi}{6} + \pi k, k \in I$

Solutions in the domain $[0, 2\pi)$ are: ___________________________________.

Example 3: Solving Trigonometric Equations

Solve for θ in general form and then in the specified domain. Use exact solutions when possible.

a. $\sec 3\theta = \frac{3}{2}, 0 \le \theta \le 360°$

b. $\sqrt{3}\csc(\theta + 20°) + 2 = 0, -180° \le \theta < 360°$

c. $3\tan\left(\theta - \frac{\pi}{4}\right) = -\sqrt{3}, 0 \le \theta \le 3\pi$

d. $16 = 8\cot\frac{1}{3}\theta, -4\pi \le \theta \le 4\pi$

e. $\sin\left(\frac{1}{2}\theta - 40°\right) = 0, -720° \le \theta < 720°$

f. $\sqrt{2}\cos[2(\theta + 1)] + 1 = 0, -\pi \le \theta \le \pi$

Solution:

$\sec 3\theta = \frac{3}{2}, 0 \le \theta \le 360°$
$\sqrt{3}\csc(\theta + 20°) + 2 = 0, -180° \le \theta < 360°$

$3\tan\left(\theta-\frac{\pi}{4}\right)=-\sqrt{3}, 0\le\theta\le3\pi$
$16=8\cot\frac{1}{3}\theta, -4\pi\le\theta\le4\pi$
$\sin\left(\frac{1}{2}\theta-40^\circ\right)=0, -720^\circ\le\theta<720^\circ$
$\sqrt{2}\cos[2(\theta+1)]+1=0, -\pi\le\theta\le\pi$

Example 4: Solving a Trigonometric Equation Application

The London Eye is a huge Ferris wheel with diameter 135 meters (443 feet) in London, England, which completes one rotation every 30 minutes.
The height of a rider on the London Eye Ferris wheel can be determined by the equation

$$h(t) = -67.5\cos\left(\frac{\pi}{15}t\right) + 69.5$$

where h(t) is the height in metres and t is the time in minutes. How long is the rider more than 100 metres above ground?

Solution:

Extra Practice:

The depth of water at a dock rises and falls with the tide, following the equation $f(t) = 4\sin\left(\frac{\pi}{12}t\right) + 7$, where t is measured in hours after midnight. A boat requires a depth of 9 feet to come to the dock. At what times will the depth be 9 feet?