Equations & Graphs of Polynomial Functions

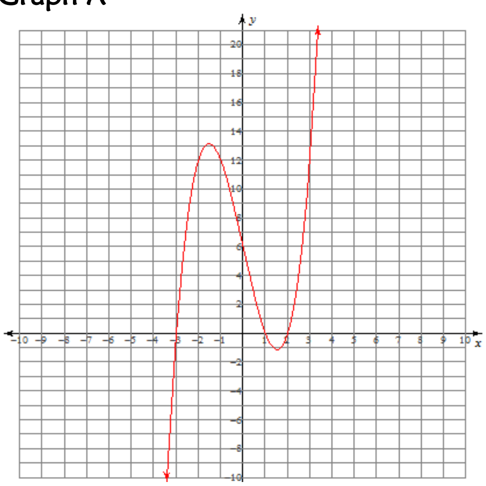
The zeros of any polynomial function y = f(x) correspond to the x-intercepts of the graph and to the roots of the corresponding equation, f(x)=0. For example, the zeroes of the function f(x) = (x- 2)(x+3)(x-5) are x=2, -3 and 5. Thus the x-intercepts of the graph of the function will be at 2, -3 and 5. These will also be the roots of the corresponding equation (x-2)(x+3)(x-5)=0.

Example 1: Analyze Graphs of Polynomial Functions

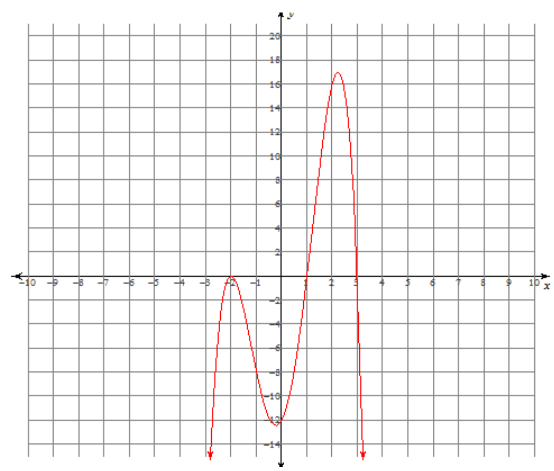
For each graph of a polynomial function shown, determine:

a. the least possible degree of the function
b. the sign of the leading coefficient
c. the x-intercepts and the factors of the function with least possible degree
d. the intervals where the function is positive and the intervals where it is negative

Graph A



Graph B



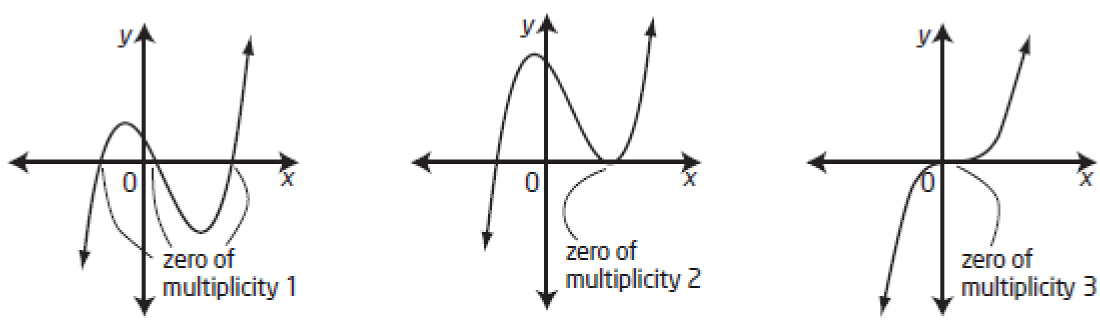
Solution:

Graph A	Graph B
a. the least possible degree ___________	a. the least possible degree ___________
b. the sign of the leading coefficient ______	b. the sign of the leading coefficient ______
c. the x-intercepts factors of the function	c. the x-intercepts factors of the function
d. intervals where the function is positive intervals where it is negative	d. intervals where the function is positive intervals where it is negative

Multiplicity of a Zero

- The number of times a zero of a polynomial function occurs. For example, the function $f(x) = (x+3)^2(x-1)$ has a zero of multiplicity 2 at x =–3.
- The multiplicity of a zero or root can also be referred to as the order of the zero or root.
- The shape of the graph of a polynomial function close to a zero of x = a (multiplicity n) is similar to the shape of the graph of a function with degree equal to n of the form $y = (x-a)^n$. For example, the graph of a function with a zero of x=1 (multiplicity 3) will look like the graph of the graph of the cubic function (degree 3) $y = (x-1)^3$ in the region close to x=1.
- Polynomial functions change sign at x-intercepts that correspond to odd multiplicity. The graph crosses over the x-axis at these intercepts.
- Polynomial functions do not change sign at x-intercepts of even multiplicity. The graph touches, but does not cross, the x-axis at these intercepts.

To illustrate this concept, consider the following graphs of degree 3 polynomial functions:



Sketching Graphs of Polynomial Functions

To sketch the graph of a polynomial function, determine characteristics such as:

- the degree of the function
- the sign of the leading coefficient
- end behaviour
- the y-intercept
- the x-intercepts
- intervals where the graph is positive / negative
- other points on the graph

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Example 2: Analyze Equations to Sketch Graphs of Polynomial Functions

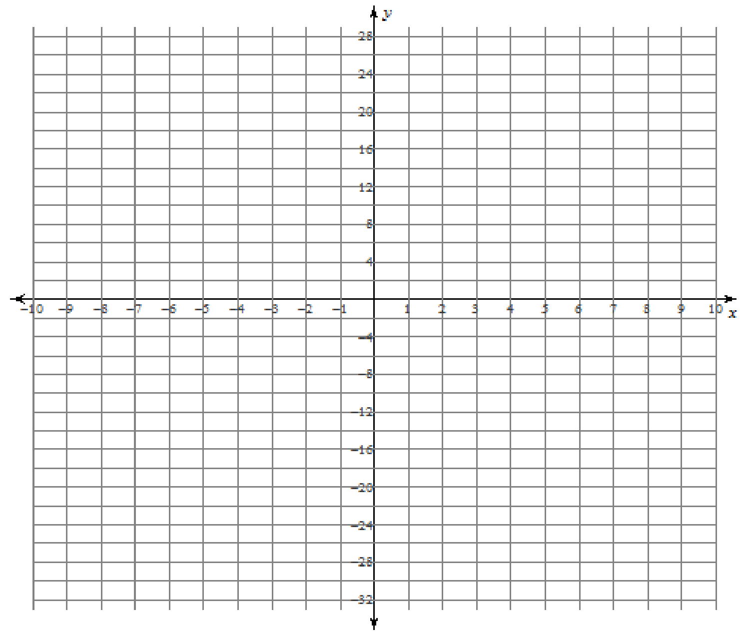
Sketch the graph of each polynomial function.

a. $f(x)=(x+4)(x+2)(x-3)$ b. $f(x)=-(x-1)^3(x+3)$ c. $f(x)=-2x^3+6x-4$

Solution:

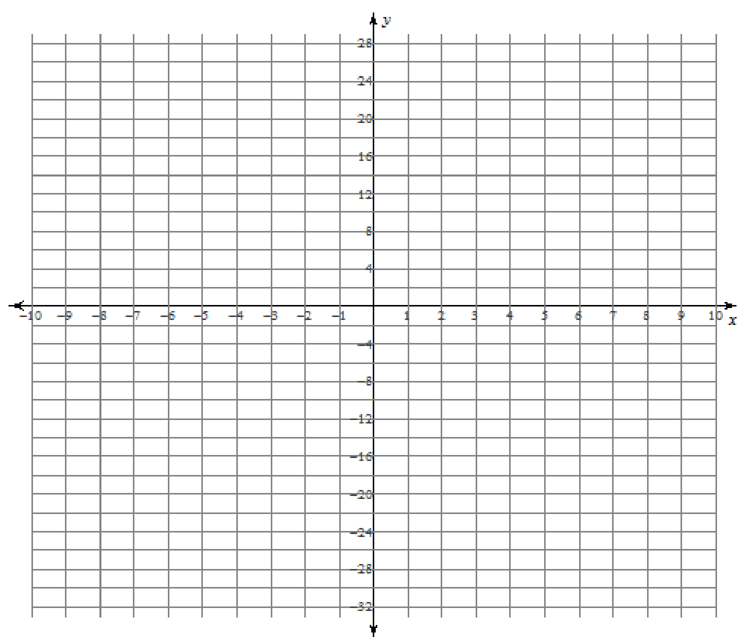
a. $f(x)=(x+4)(x+2)(x-3)$

Degree	
Leading Coefficient	
End Behaviour	
Zeros/x-intercepts	
y-intercept	
Interval(s) where the function is positive or negative	
Other points	



b. $f(x)=-(x-1)^3(x+3)$

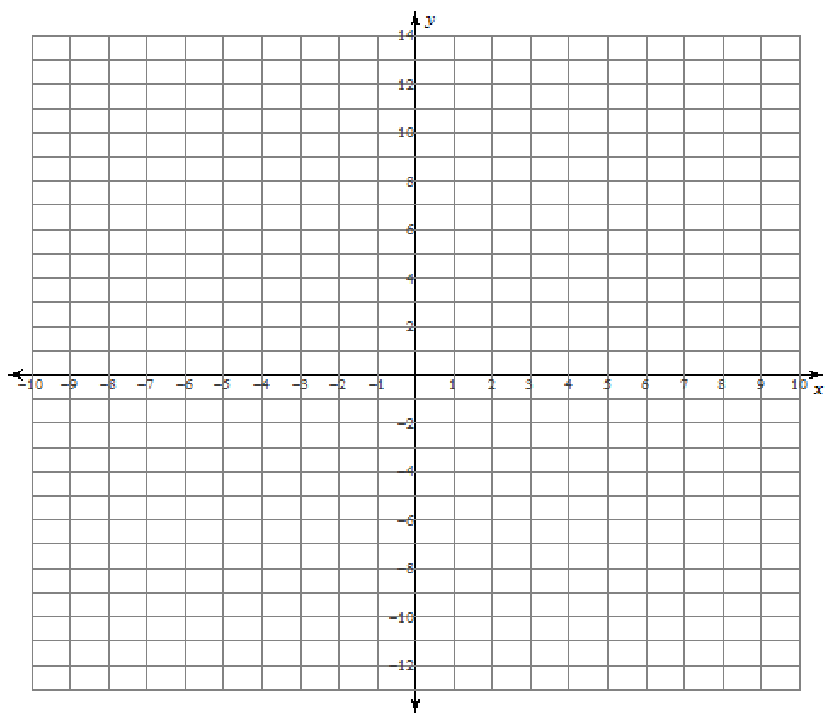
Degree	
Leading Coefficient	
End Behaviour	
Zeros/x-intercepts	
y-intercept	
Interval(s) where the function is positive or negative	
Other points	



c. $f(x) = -2x^3 + 6x - 4$

Determine the factored form of the function:

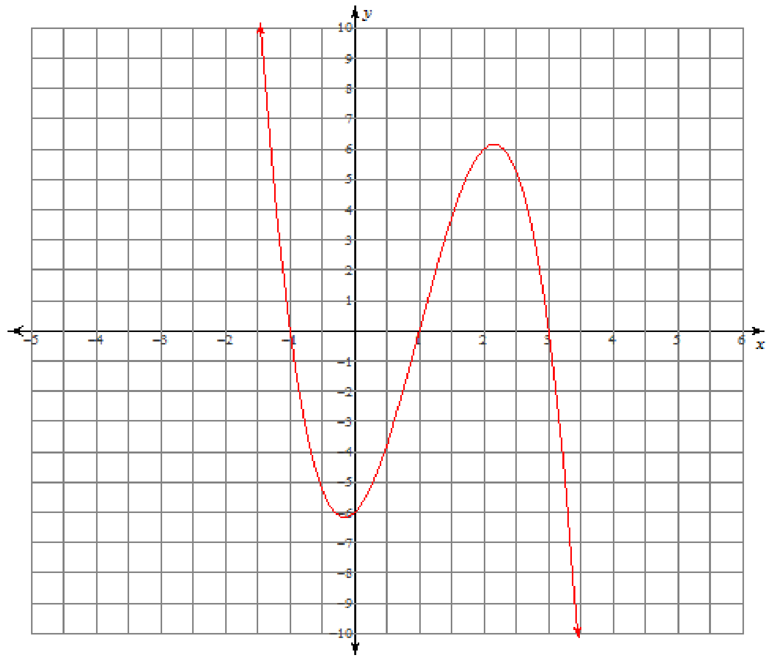
Degree	
Leading Coefficient	
End Behaviour	
Zeros/x-intercepts	
y-intercept	
Interval(s) where the function is positive or negative	
Other points	



Example 3: Determining the Equation of a Polynomial Function From Its Graph

Use the graph of the given function to write the corresponding polynomial equation. State the roots of the equation. The roots are all integral values.

Solution:



The graph of the function has _______x –intercepts. All of the x-intercepts are of even/odd multiplicity. The least possible multiplicity of each x-intercept is __________, so the least possible degree is _____________.

The graph extends up into quadrant ______ and down into quadrant ______, so the leading coefficient is positive /negative.

The y-intercept is ________; this is the _______________ term in the equation of the function.

The zeroes, or x-intercepts, are ______, ______, and ______. The product of the roots is _________.

Compare the product of the roots to the y-intercept to determine the vertical stretch, a.

a= _______

The equation of the polynomial function is

f(x)=________(________)(________)(________)

Graphing Polynomial Functions Using Transformations

The graph of a function of the form $y = a(b(x-h))^n + k$ is obtained by applying transformations to the graph of the general polynomial function $y = x^n$, where $n \in N$ .The effects of changing parameters in polynomial functions are the same as the effects of changing parameters in other types of functions. Remember to apply the reflections and stretches before the translations.

Example 4: Apply Transformations to Sketch a Graph

The graph of $y = x^3$ is transformed to obtain the graph of $y = \frac{1}{5}(2(x+4))^3 - 5$.

a. Describe the resulting transformations.

b. Sketch the graph of $y = \frac{1}{5}(2(x+4))^3 - 5$

Solution:

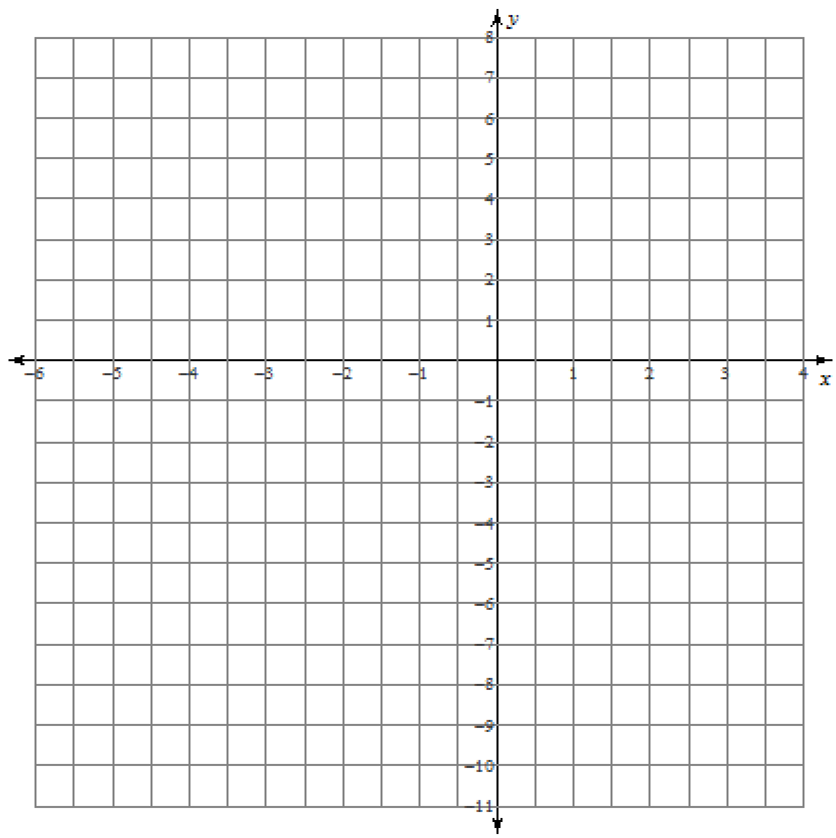
a. Transformations:

b. Sketch the graph of $y = \frac{1}{5}(2(x+4))^3 - 5$

Mapping Rule: ____________________________

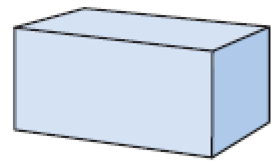
$y = x^3$	
x	y
-3	
-2	
-1	
0	
1	
2	
3	

$y = \frac{1}{5}(2(x+4))^3 - 5$	
x	y



Example 5: Model and Solve Problems Involving Polynomial Functions

Alicia is preparing to make an ice sculpture. She has a block of ice that is 4 ft wide, 5 feet long and 6 ft high. Alicia wants to reduce the size of the block of ice by removing the same amount from each of the three dimensions. She wants to reduce the volume of the ice block to $24ft^3$.



a. Write a equation that models this situation.
b. How much should she remove from each dimension?

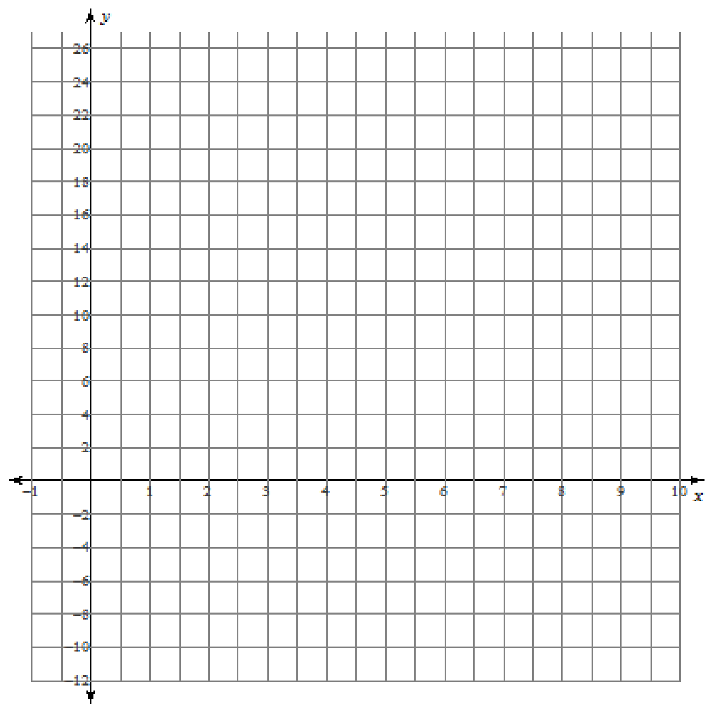
Solution:

a. V(x)= ________________________________= ______________________

b. How much should Alicia remove from each dimension?

Method 1: Graphical Solution

Enter the equations y_1 = ______________________ and y_2 = ________ into the graphing calculator and determine the x-coordinate of the point of intersection of the two graphs.



_______ **feet must be removed from the block of ice.**

Method 2: Factoring

__________ **ft must be removed from the block of ice.**