# The Remainder Theorem

## Long Division

You can use long division to divide a polynomial by a binomial: $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$

The components of long division are

- the dividend, P(x), which is the polynomial that is being divided
- the divisor, x-a, which is the binomial that the polynomial is being divided by
- the quotient, Q(x), which is the expression that results from the division
- the remainder, R, which is the value or expression that is left over after dividing

To check the division of a polynomial, verify the statement $P(x) = (x-a)Q(x) + R$. In other words, multiply the quotient, Q(x), by the divisor, x-a, and add the remainder, R, to the product. The result is the dividend, P(x).

## Example 1: Divide a Polynomial by a Binomial of the Form x-a

a. Divide $P(x) = 9x + 4x^3 - 12$ by $x+2$. Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$
b. Identify any restrictions on the variable.
c. Write the corresponding statement that can be used to check the division.

**Solution:**

**a.** $x+2 \overline{\smash{)}4x^3 + 0x^2 + 9x - 12}$

**b.** Restrictions on the variable

**c.** $P(x) = (x-a)Q(x) + R$

## Example 2: Apply Polynomial Division to Solve a Problem

The volume, V, in cubic centimeters, of gift boxes is given by $V(x) = 2x^3 + x^2 - 27x - 36$. The height, h, in centimeters is x+3. What are the possible dimensions of the boxes in terms of x?

**Solution:**

Divide the _______________ of the box by the _______________ to obtain an expression for the _______________ of the base of the box. Then, factor this expression to obtain expressions for the _______________ and _______________ of the base.

**Expressions for the dimensions, in centimeters, are _____________, ________________, _______________.**

## Synthetic Division

- a short form of division that uses only the coefficients of the terms and fewer calculations.

## Example 3: Divide a Polynomial Using Synthetic Division

a. Use *long division* to divide $5x^2 - x + 2x^3 - 6$ by x + 2.
b. State the restriction.
c. Use *synthetic division* to divide $5x^2 - x + 2x^3 - 6$ by $x + 2$..

**Solution:**

# Remainder Theorem

The **remainder theorem** states that when a polynomial in x, P(x), is divided by a binomial of the form x-a, the remainder is P(a).

- If the remainder is 0, then the binomial x-a is a factor of P(x)
- If the remainder is not 0, then the binomial x-a is not a factor of P(x).

## Example 4: Apply the Remainder Theorem

**a.** Use the remainder theorem to determine the remainder when $P(x) = 3x^4 - x^3 - 5$ is divided by x-3.
**b.** Verify your answer using long division.
**c.** Verify your answer using synthetic division.

**Solution:**

**a.** $P(x) = 3x^4 - x^3 - 5$

**b.**

**c.**