Combinations

Combination: a selection of objects without regard to order.
Examples of combinations include choosing five fruits from a possible eight to make a fruit salad, choosing four people from a group of ten to serve on a committee, picking three colors from a color brochure.

"My fruit salad is a combination of apples, grapes and bananas" . We don't care what order the fruits are in, they could also be "bananas, grapes and apples" or "grapes, apples and bananas", it is the same fruit salad.

Formula for Combinations

The FHS Physical Education department has 3 free passes to a school hockey game to give away. Anna, Brandon, Chloe, Derek and Emily are all interested in having one of the free passes. How many ways can the Phys Ed department give away the passes?

First determine how many ways you can select three people in a particular order from a group of 5.

Number of choices for 1st pass	Number of choices for the 2nd pass	Number of choices for the 3rd pass
5	4	3

There are (5)(4)(3)=60 or ${}_5P_3$ ways to select 3 people from a group of 5, if the order matters. But consider the following selections of three people from the group of five.

ABC ACB BAC BCA CAB CBA

These groups represent six of the sixty possible permutations of three students from five, but they represent only one possible combination of three students from five. Similarly, every other possible combination of three from five would be represented in 6 different ways as permutations. So, the 60 permutations would have to be reduced by a factor of 6, or 3!, to obtain the number of possible combinations:

$$\frac{{}_5P_3}{3!} = \frac{\frac{5!}{2!}}{3!} = \frac{5!}{2!3!} = {}_5C_3 = 10$$

The notation, ${}_nC_r$ is used to represent the number of combinations, without regard to order, of r items taken from a set of n distinct items.

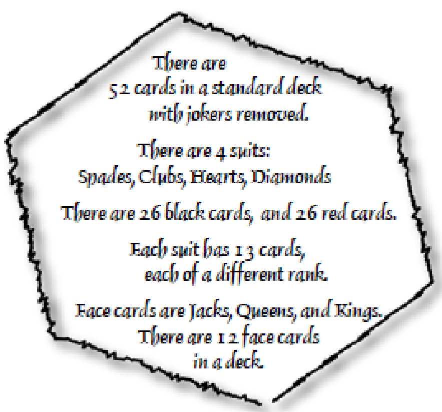
FORMULA FOR COMBINATIONS: $${}_nC_r = \frac{{}_nP_r}{r!} = \frac{\frac{n!}{(n-r)!}}{r!} = \frac{n!}{r!(n-r)!}$$

Example 1: Basic Combinations

A pizza can have 3 toppings out of a possible 7 toppings. How many different pizzas's can be made?

Solution:

Example 2: Combinations Including Specific Items



From a deck of 52 cards, a 5 card hand is dealt. How many distinct five card hands are there if the queen of spades and the four of diamonds must be in the hand?

Solution:

Example 3: Combinations From Multiple Selection Pools

a. A committee of 3 boys and 5 girls is to be formed from a group of 10 boys and 11 girls. How many committees are possible?

b. From a deck of 52 cards, a 7 card hand is dealt. How many distinct hands are there if the hand must contain 2 spades and 3 diamonds?

Solution:

a. A committee of 3 boys and 5 girls is to be formed from a group of 10 boys and 11 girls. How many committees are possible?

b. From a deck of 52 cards, a 7 card hand is dealt. How many distinct hands are there if the hand must contain 2 spades and 3 diamonds?

Example 4: At Least/At Most

A research team of 5 members is to be formed from a selection pool of 8 chemists and 9 biologists. How many research teams will have

a. At least 3 biologists
b. At least 1 chemist
c. At most 2 chemists

Solution:

a. At least 3 biologists

b. At least 1 chemist

c. At most 2 chemists

Example 5: Simplifying Expressions and Solving Equations With Combinations

a. Express as factorials and simplify $\frac{{}_{n}C_{7}}{{}_{n-1}C_{5}}$

b. Solve for n if $3({}_{n}C_{3}) = {}_{n+1}C_{4}$

Solution:

a.

b.