4.4 Fractional Exponents and Radicals

Use a calculator to complete the table.

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} =$
4	$4^{\frac{1}{2}} =$
9	
16	
25	

Use a calculator to complete the table.

x	$x^{\frac{1}{3}}$
1	
8	
27	
64	
125	

What do you think the fractional exponents mean?

What do you think $a^{\frac{1}{4}}$ and $a^{\frac{1}{5}}$ mean?

What does $a^{\frac{1}{n}}$ mean? Explain your reasoning.

Powers with Rational Exponents with Numerator 1

When n is a natural number and x is a rational number, $x^{\frac{1}{n}} = \sqrt[n]{x}$

Example 1 Evaluating Powers of the Form $a^{\frac{1}{n}}$

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$ b) $0.49^{\frac{1}{2}}$ c) $(-64)^{\frac{1}{3}}$ d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

SOLUTION

1. Evaluate each power without using a calculator.

a) $1000^{\frac{1}{3}}$ b) $0.25^{\frac{1}{2}}$

c) $(-8)^{\frac{1}{3}}$ d) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number,

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m \quad \text{and} \quad x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

Example 2 Rewriting Powers in Radical and Exponent Form

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.

b) Write $\sqrt{3^5}$ and $\left(\sqrt[3]{25}\right)^2$ in exponent form.

Example 3 Evaluating Powers with Rational Exponents and Rational Bases

Evaluate.

a) $0.04^{\frac{3}{2}}$ b) $27^{\frac{4}{3}}$ c) $(-32)^{0.4}$ d) $1.8^{1.4}$

Example 4 Applying Rational Exponents

Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass m kilograms. Estimate the brain mass of each animal.

a) a husky with a body mass of 27 kg

b) a polar bear with a body mass of 200 kg