Limits 1: Homework Solutions.

(a) $\lim\limits_{x\to 4} \dfrac{x-4}{x^3-64}$ ← substitution gives $\frac{0}{0}$ so try factoring...

→ difference of cubes: $a = x$ $d = 4$: $(a-d)(a^2+ad+d^2)$

$$\lim_{x\to 4} \frac{x-4}{(x-4)(x^2+4x+16)}$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}}{\cancel{(x-4)}(x^2+4x+16)} = \lim_{x\to 4} \frac{1}{(x^2+4x+16)} = \frac{1}{(16)+(16)+(16)} = \boxed{\frac{1}{48}}$$

(b) $\lim\limits_{x\to 7} \dfrac{\sqrt[5]{3-5x}}{(x-5)^3}$ ← try substitution

$$= \frac{\sqrt[5]{3-5(7)}}{(7-5)^3} = \frac{\sqrt[5]{-32}}{2^3} = \frac{-2}{8} = \boxed{\frac{-1}{4}}$$

(c) $\lim\limits_{x\to -3} \dfrac{x+3}{3-\sqrt{x+12}}$ ← substitution results in $\frac{0}{0}$

← factor? No

← so try multiply by conjugate...

$$\lim_{x\to -3} \frac{(x+3)}{(3-\sqrt{x+12})} \frac{(3+\sqrt{x+12})}{(3+\sqrt{x+12})} = \lim_{x\to -3} \frac{(x+3)(3+\sqrt{x+12})}{9-(x+12)}$$

$$= \lim_{x\to -3} \frac{(x+3)(3+\sqrt{x+12})}{9-x-12} = \lim_{x\to -3} \frac{(x+3)(3+\sqrt{x+12})}{(-x-3)}$$

$$= \lim_{x\to -3} \frac{(x+3)(3+\sqrt{x+12})}{-(x+3)} = \lim_{x\to -3} \frac{\cancel{(x+3)}(3+\sqrt{x+12})}{-\cancel{(x+3)}}$$

$$= \lim_{x\to -3} \frac{3+\sqrt{x+12}}{-1} = \frac{3+\sqrt{-3+12}}{-1} = \frac{3+\sqrt{9}}{-1} = \frac{3+3}{-1} = \boxed{-6}$$

(d) $\lim_{x \to -3} \sqrt[3]{\dfrac{x-4}{6x^2+2}}$ ← Try substitution?

$$\lim_{x \to -3} \sqrt[3]{\frac{-3-4}{6(-3)^2+2}} = \sqrt[3]{\frac{-7}{6(9)+2}} = \sqrt[3]{\frac{-7}{56}} = \sqrt[3]{\frac{-1}{8}} = \frac{-1}{2}$$

(e) $\lim_{x \to 3} \left(\dfrac{\frac{1}{x^2} - \frac{1}{9}}{x-3} \right)$ ← substition gives $\frac{0}{0}$

← can't factor the way it is ~~so~~ try to simplify...

$\lim_{x \to 3} \dfrac{\left[\frac{1}{x^2} - \frac{1}{9}\right]}{(x-3)}$ ← on numerator, get common denominator of $9x^2$

$$\text{so: } \lim_{x \to 3} \frac{\left[\frac{9}{9x^2} - \frac{x^2}{9x^2}\right]}{(x-3)} = \lim_{x \to 3} \frac{\left[\frac{9-x^2}{9x^2}\right]}{(x-3)} = \lim_{x \to 3} \frac{\left[\frac{-x^2+9}{9x^2}\right]}{(x-3)}$$

Multiply by reciprocal

$$= \lim_{x \to 3} \frac{\left[\frac{-(x^2-9)}{9x^2}\right]}{\left[\frac{x-3}{1}\right]} = \lim_{x \to 3} \left[\frac{-(x-3)(x+3)}{9x^2}\right] * \left[\frac{1}{x-3}\right]$$

$$\lim_{x \to 3} \frac{-\cancel{(x-3)}(x+3)}{(9x^2)\cancel{(x-3)}} = \lim_{x \to 3} \frac{-(x+3)}{9x^2} = \frac{-6}{9(9)} = \frac{-6}{81} = \boxed{\frac{-2}{27}}$$

(f) $\lim_{h \to 0} \dfrac{(-3+h)^2 - 9}{h}$ ← FACTOR: Consider numerator a difference of squares:

$(a^2 - d^2) = (a-d)(a+d)$

$= \lim_{h \to 0} \dfrac{((-3+h)-3)((-3+h)+3)}{h}$ where $a = (-3+h)$ and $d = 3$

$$= \lim_{h \to 0} \frac{(-6+h)(h)}{h} = \lim_{h \to 0} \frac{(-6+h)\cancel{(h)}}{\cancel{(h)}} = \lim_{h \to 0} (-6+h) = -6+0 = \boxed{-6}$$

Limits 1: (con't)

(g) $\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$ ← factor: consider the numerator a difference of cubes where $a = (2+h)$ $d = 2$

: $(a-d)(a^2 + ad + d^2)$

$$= \lim_{h \to 0} \frac{((2+h)-2)((2+h)^2 + (2+h)(2) + 4)}{h}$$

$$= \lim_{h \to 0} \frac{(h)((2+h)^2 + 4 + 2h + 4)}{(h)} = \lim_{h \to 0} \frac{\cancel{(h)}((2+h)^2 + 8 + 2h)}{\cancel{(h)}}$$

$$= \lim_{h \to 0} (2+h)^2 + 8 + 2h = (2+0)^2 + 8 + 2(0) = 4 + 8 = \boxed{12}$$

(h) $\lim_{x \to 0} \frac{\left[\frac{1}{1+x} - 1\right]}{x}$ ← similar to (e) – get a common denominator

$$\lim_{x \to 0} \frac{\frac{1}{1+x} - \frac{1+x}{1+x}}{x} = \lim_{x \to 0} \frac{\left[\frac{1-1-x}{1+x}\right]}{\left[\frac{x}{1}\right]} = \lim_{x \to 0} \left[\frac{-x}{1+x}\right] * \left[\frac{1}{x}\right]$$

$$= \lim_{x \to 0} \left(\frac{-\cancel{x}}{1+x}\right)\left(\frac{1}{\cancel{x}}\right) = \lim_{x \to 0} \frac{-1}{1+x} = \frac{-1}{1+0} = \boxed{-1}$$

(i) $\lim_{x \to -1} \frac{2x+3}{3x+2}$ ← substitution works!

$$= \frac{2(-1)+3}{3(-1)+2} = \frac{-2+3}{-3+2} = \frac{1}{-1} = \boxed{-1}$$

Limits 1: (con't)

(j) $\lim_{x \to -8} \frac{x+8}{\sqrt[3]{x}+2}$ ← a tricky one! For this one, consider $x+8$ being a sum of cubes and factor as $a = \sqrt[3]{x}$ and $d = 2$

Remember: $(a+d)(a^2 - ad + d^2)$

$$\lim_{x \to -8} \frac{(\sqrt[3]{x}+2)((\sqrt[3]{x})^2 - 2\sqrt[3]{x} + 4)}{(\sqrt[3]{x}+2)}$$

$$= \lim_{x \to -8} \frac{\cancel{(\sqrt[3]{x}+2)}((\sqrt[3]{x})^2 - 2\sqrt[3]{x} + 4)}{\cancel{(\sqrt[3]{x}+2)}} = \lim_{x \to -8} ((\sqrt[3]{x})^2 - 2\sqrt[3]{x} + 4)$$

$$= ((\sqrt[3]{-8})^2 - 2\sqrt[3]{-8} + 4) = (-2)^2 - 2(-2) + 4 = 4+4+4 = \boxed{12}$$

(k) $\lim_{x \to -1} \frac{8}{(3+x)\sqrt{3-x}}$ ← substitution

$$= \frac{8}{(3+(-1))\sqrt{3-(-1)}} = \frac{8}{(2)\sqrt{4}} = \frac{8}{2 \cdot 2} = \frac{8}{4} = \boxed{2}$$

(l) $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$ – multiply num. & den. by conj. of den.

$$= \lim_{x \to 9} \frac{(x-9)}{(3-\sqrt{x})} \cdot \frac{(3+\sqrt{x})}{(3+\sqrt{x})} = \lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{9-x} = \lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{-(x-9)}$$

$$= \lim_{x \to 9} \frac{\cancel{(x-9)}(3+\sqrt{x})}{-\cancel{(x-9)}} = \lim_{x \to 9} \frac{3+\sqrt{x}}{-1} = \frac{3+\sqrt{9}}{-1} = \frac{3+3}{-1} = \boxed{-6}$$

Limits I : (con't)

(m) $\lim\limits_{x \to 0} \dfrac{(3+x)^3 - 3^3}{(3+x)^2 - 3^2}$ — factor as diff of cubes: $a=3+x,\ d=3$
— factor as diff. of squares $a=3+x$ $d=3$

$$\lim_{x \to 0} \frac{((3+x)-3)((3+x)^2 + (3+x)(3) + 3^2)}{((3+x)-3)((3+x)+3)}$$

$$= \lim_{x \to 0} \frac{(x)((3+x)^2 + 9 + 3x + 9)}{(x)(6+x)} = \lim_{x \to 0} \frac{(3+x)^2 + 3x + 18}{(6+x)}$$

$$= \frac{((3)^2 + 0 + 18)}{(6+0)} = \frac{9+18}{6} = \frac{27}{6} = \boxed{\frac{9}{2}}$$

(n) $\lim\limits_{x \to 9} \dfrac{x^2 - 81}{3 - \sqrt{x}}$ → Multiply by conjugate of denominator

$$\lim_{x \to 9} \frac{(x^2-81)}{(3-\sqrt{x})} \cdot \frac{(3+\sqrt{x})}{(3+\sqrt{x})} = \lim_{x \to 9} \frac{(x-9)(x+9)(3+\sqrt{x})}{(9-x)}$$

$$= \lim_{x \to 9} \frac{(x-9)(x+9)(3+\sqrt{x})}{-(x-9)} = \lim_{x \to 9} \frac{(x+9)(3+\sqrt{x})}{-1} = \frac{(9+9)(3+\sqrt{9})}{-1}$$

$$= \frac{(18)(3+3)}{-1} = \frac{18(6)}{-1} = \boxed{-108}$$

(o) $\lim\limits_{x \to 16} \dfrac{2\sqrt{x} + x^{3/2}}{\sqrt[4]{x} + 5}$ → Substitution

$$= \frac{2\sqrt{16} + 16^{3/2}}{\sqrt[4]{16} + 5} = \frac{2(4) + 64}{2+5} = \frac{8+64}{7} = \boxed{\frac{72}{7}}$$

→ FACTOR:

p) $\lim_{x\to 2} \frac{x^2-4}{3x^2-7x+2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{(3x-1)\cancel{(x-2)}} = \lim_{x\to 2} \frac{(x+2)}{(3x-1)} = \frac{4}{5}$

: Conjugate

q) $\lim_{x\to 7} \frac{(\sqrt{x+2}-3)}{(x-7)}\frac{(\sqrt{x+2}+3)}{(\sqrt{x+2}+3)} = \lim_{x\to 7} \frac{\overset{(x-7)}{((x+2)-9)}}{\cancel{(x-7)}(\sqrt{x+2}+3)} = \lim_{x\to 7} \frac{1}{\sqrt{x+2}+3} = \frac{1}{6}$

r) $\lim_{x\to 4} \underbrace{\left(\frac{8}{x^2-16} - \frac{1}{x-4}\right)}_{\text{· get a common denominator}} = \lim_{x\to 4} \frac{8-1(x+4)}{(x-4)(x+4)} = \lim_{x\to 4} \frac{8-x-4}{(x-4)(x+4)} = \lim_{x\to 4} \frac{-x+4}{(x-4)(x+4)} = \lim_{x\to 4} \frac{-(x-4)}{(x-4)(x+4)} = \lim_{x\to 4} \frac{-1}{(x+4)}$

$= \frac{-1}{4+4} = -1/8$

s) $\lim_{x\to 5} \frac{2x}{x-5}$

$\lim_{x\to 5^+} \frac{2x}{x-5} = \frac{2(5.001)}{(5.001-5)} = \frac{10.002}{0.001} = 10002$

Because these are not equal... DNE

$\lim_{x\to 5^-} \frac{2x}{x-5} = \frac{2(4.999)}{4.999-5} = \frac{9.998}{-0.001} = -1998$

→ only way to do this one is use values very close to $x=5$: on left ($x=4.999$) and right ($x=5.001$)

t) $\lim_{x\to 0} \frac{x^3}{2x^3+3x^4} = \lim_{x\to 0} \frac{\cancel{x^3}}{\cancel{x^3}(2+3x)} = \lim_{x\to 0} \frac{1}{(2+3x)} = \frac{1}{2}$

FACTOR

Factor:

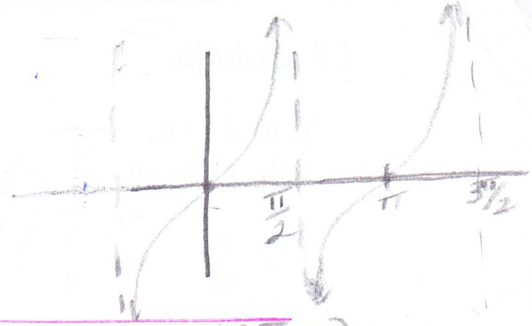
u) $\lim_{x\to 2} \frac{x^2-2x}{2x^2-7x+6} = \lim_{x\to 2} \frac{x\cancel{(x-2)}}{(2x-3)\cancel{(x-2)}} = \lim_{x\to 2} \frac{x}{2x-3} = \frac{2}{1}$

v) $\lim_{x\to\pi} \frac{1+\sin x}{\cos x} = \frac{1+0}{-1} = -1$

Substitution:

w) $\lim_{x \to \frac{\pi}{4}} ((\cos x)+(\sin x)) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$

Remember tanθ graph:



x.) $\lim_{x \to \frac{3\pi}{2}} \frac{\sin x}{(\cos x)} = \lim_{x \to \frac{3\pi}{2}} \tan x = \boxed{DNE}$

(y) $\lim_{x \to 4} \frac{2x^3 - 128}{\sqrt{x} - 2} \to \lim_{x \to 4} \frac{2(x^3 - 64)}{(\sqrt{x} - 2)} \cdot \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} \to \lim_{x \to 4} \frac{2(x^3-64)(\sqrt{x}+2)}{x-4}$

$\lim_{x \to 4} \frac{2(x-4)(x^2+4x+16)(\sqrt{x}+2)}{(x-4)} \to \lim_{x \to 4} 2(x^2+4x+16)(\sqrt{x}+2)$

$= 2(16+16+16)(4)$

$= 2(48)(4)$

$= \boxed{384}$

Factor then conjugate, the factor.