Derivative of the Sine Function

For $f(x) = \sin x$ find an expression for $f'(x)$.

Derivative of the Cosine Function

For $f(x) = \cos x$

$$\frac{d}{dx}(\cos x) = -\sin x$$

the derivation is left for you to try in practice problem # 24.

EXAMPLE 1: Revisiting the Differentiation Rules

Find the derivatives of

a) $y = 3x^2 \sin x$ b) $y = \dfrac{\sin x}{1 - \cos x}$

Simple Harmonic Motion

The motion of a weight bobbing up and down on the end of a spring is an example of **simple harmonic motion**. Example 2 describes a case in which there are no opposing forces to slow down the motion (undamped).

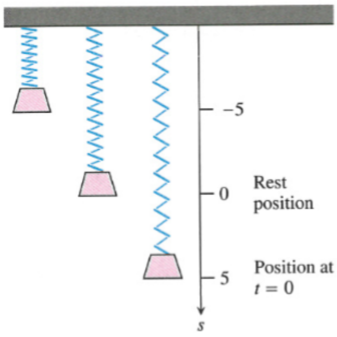
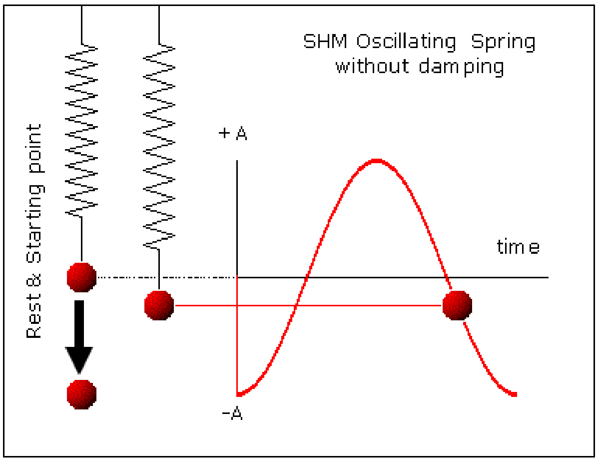


Figure 3.38 The weighted spring in Example 2.

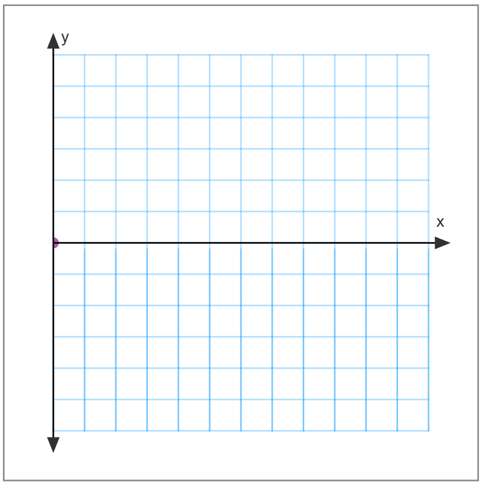


Example 2:

A weight hanging from a spring is stretched 5 units beyond its rest position ($s = 0$) and released at time $t = 0$ to bob up and down. Its position at any time later t is

$$s = 5\cos t$$

What are its velocity and acceleration at time t? Describe its motion.



Jerk

A sudden change in acceleration is called a "jerk." When a ride in a car or a bus is jerky, it is not that the accelerations are necessarily large, but that the changes in acceleration are abrupt. Jerk is what spills your coffee. All over your lap. Ugh. Stupid jerk.

DEFINITION Jerk
Jerk is the derivative of acceleration. If a body's position at time t is $s(t)$, the body's jerk at time t is

$$j(t) = \frac{da}{dt} = \frac{d^3s}{dt^3}$$

EXAMPLE 3: A Couple of Jerks

a) The jerk caused by the constant acceleration of gravity is zero:

This explains why we don't experience motion sickness while just sitting around.

b) The jerk of the simple harmonic motion in Example 2 is:

Derivatives of the Other Basic Trigonometric Functions
Because sin*x* and cos*x* are differentiable functions of *x*, the related functions

$$\tan x = \frac{\sin x}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{\cos x}{\sin x} \qquad \sec x = \frac{1}{\cos x}$$

are differentiable at every value of*x* for which they are defined. The derivations for these are left for you to try in practice problems # 25 & 26.

$$\frac{d}{dx}\tan x = \sec^2 x, \qquad \frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x, \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

EXAMPLE 4: A Trigonometric Second Derivative
Find y'' if y = sec*x*

L'Hopital's Rule for solving limits

THEOREM L'Hôpital's Rule
Suppose that $f(a) = g(a) = 0$, that $f'(a)$ and $g'(a)$ exist, and that $g'(a) \neq 0$. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} = \frac{f'(a)}{g'(a)}$$

Example 5:

Use l'Hopital's Rule to solve the following limits:

a) $\lim_{x \to 0} \frac{\sin x}{x}$

b) $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$