INSTANTANEOUS RATES OF CHANGE

We will revisit some applications in which derivatives as functions are used to represent the rates at which things change in the world around us – and see new applications as well. We have already seen that one use for the derivative is finding the *instantaneous rate of change* of a function.

EXAMPLE 1:

An orange farmer currently has 200 trees yielding an average of 15 bushels of oranges per tree. She is expanding her farm at the rate of 15 trees per year, while improving her average annual yield by 1.2 bushels per tree. What is the current (instantaneous) rate of increase of her total annual production of oranges?

<u>EXAMPLE 2</u> Enlarging Circles
a) Find the rate of change of the area A of a circle with respect to its radius r.
b) Evaluate the rate of change of A at $r = 5$ and at $r = 10$.
c) If r is measured in centimetres and A is measured in square centimetres, what units would be appropriate for dA/dr?

Motion Along a Line:

Suppose that an object is moving along a line (say an s-axis) so that we know its position s on that line as a function of time t:

$$s = f(t)$$

We have already seen that the **average velocity** of the object can be calculated this way:

$$v_{ave} = \frac{\Delta s}{\Delta t} = \frac{f(t+h) - f(t)}{h}$$

DEFINITION Instantaneous Velocity
The **(instantaneous) velocity** is the derivative of the position function $s = f(t)$ with respect to time. At time t the velocity is

$$v(t) = \frac{ds}{dt} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

Besides telling how fast an object is moving - velocity also gives a direction. When the object is moving forward (**s is increasing**), the velocity is **positive;** when the object is moving backward (**s is decreasing**), the velocity is **negative.**

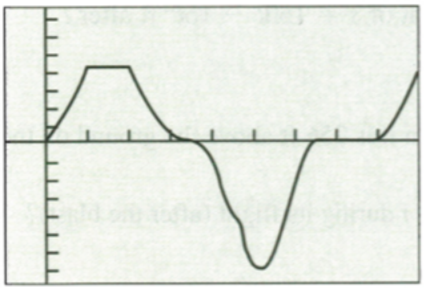
DEFINITION Speed

Speed is the absolute value of velocity:

$$speed = |v(t)| = \left|\frac{ds}{dt}\right|$$

EXAMPLE 3:

A student walks around in front of a motion detector that records her velocity at 1-second intervals for 36 seconds. She stores the data in her graphing calculator and uses it to generate the time-velocity graph shown.



Describe her motion as a function of time by reading the velocity graph. When is her *speed* a maximum?

DEFINITION Acceleration

Acceleration is the derivative of velocity with respect to time.

If a body's velocity at time t is $v(t) = \frac{ds}{dt}$, then the body's acceleration at time t is $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

EXAMPLE 4 Modeling Vertical Motion

A dynamite blast propels a heavy rock straight up with a launch velocity of 49 m/s. It reaches a height of $s = 49t - 4.9t^2$ m after t seconds.

a) How high does the rock go?

b) What is the velocity and speed of the rock when it is 78 m above the ground on the way up? the way down?

c) What is the acceleration of the rock at any time t during its flight (after the blast)?

d) When does the rock hit the ground?

EXAMPLE 5 Studying Particle Motion

A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^2 - 4t + 3$, where s is measured in metres and t is measured in seconds.

a) Find the displacement of the particle during the first 2 seconds.

b) Find the average velocity of the particle during the first 4 seconds.

c) Find the instantaneous velocity of the particle when $t = 4$.

d) Find the acceleration of the particle when $t = 4$.

e) Describe the motion of the particle. At what values of t does the particle change direction?

Derivatives in Economics:

In economics we refer to the rates of change or derivatives as *marginals.*

Here are some functions commonly dealt with in economics:

- *Cost of production*, $c(x)$; $c'(x)$ gives the *marginal cost*
- *Revenue from sales,* $r(x)$; $r'(x)$ gives the *marginal revenue*
- *Profit*, $P(x)$; $P'(x)$ gives the *marginal profit*

Since we are working with production of items depending on whole numbers, we sometimes refer to the *marginals* as the extra cost/ revenue/profit of producing one more unit.

EXAMPLE 6: Marginal Cost and Marginal Revenue

Suppose it costs $c(x)=x^3-6x^2+15x$ dollars to produce x radiators when 8 to 10 radiators are produced, and that $r(x)=x^3-3x^2+12x$ gives the dollar revenue from selling x radiators. Your shop currently produces 10 radiators a day. Find the marginal cost, marginal revenue and marginal profit for this production level.