**<u>Powers, Multiples, Sums and Differences</u>:**

**Derivative of a Constant**

If *f* is a function with the constant value *c* (a constant), then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0$$

**PROOF:**

**Power Rule for Positive Integer Powers of *x***

If *n* is a positive integer, then

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

**PROOF:**

**The Constant Multiple Rule**

If $f$ is a differentiable function of $x$ and $c$ is a constant, then

$$\frac{d}{dx}(cf) = c\frac{df}{dx}$$

**PROOF:**

**The Sum and Difference Rule**

If $f$ and $g$ are differentiable functions of $x$, then their sum and difference are differentiable at every point where $f$ and $g$ are differentiable. At such points,

$$\frac{d}{dx}(f \pm g) = \frac{df}{dx} \pm \frac{dg}{dx}$$

**PROOF:**

**Example 1:** Find f '(x) for the following functions:

a) $f(x) = x^4 - 3x^2 + 1$

b) $f(x) = \sqrt{x}$

c) $f(x) = 1 - \pi x^9$

**EXAMPLE 2:**
Does the curve $y = x^4 - 6x^2 + 5$ have any horizontal tangents? If so, where?

## PRODUCTS AND QUOTIENTS

The derivative of the product of two functions is *not* the product of their derivatives. For example,

$$\frac{d}{dx}(x \cdot x) = \frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(x) \cdot \frac{d}{dx}(x) = 1 \cdot 1 = 1$$

The derivative of a product is actually the sum of *two* products.

**The Product Rule**

The product of two differentiable functions $f(x)$ and $g(x)$ is differentiable, and

$$\frac{d}{dx}(fg) = f \cdot \frac{dg}{dx} + g \cdot \frac{df}{dx}$$

**PROOF:**

**EXAMPLE 3:**
Find $g'(x)$ if $g(x)=(x^2-2)(x^3-3x)$

**EXAMPLE 4:**
Let $y = uv$ be the product of the functions $u$ and $v$. Find $y'(2)$ if

$u(2)=3$ $u'(2)=-4$ $v(2)=1$ $v'(2)=2$

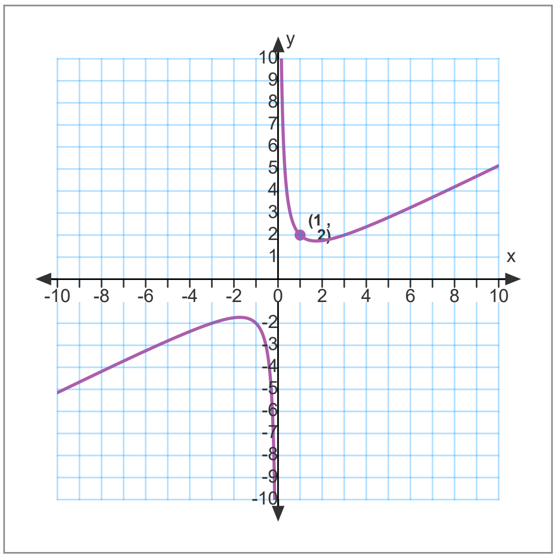
**The Quotient Rule**
At a point where $g \neq 0$, the quotient $y = f/g$ of two differentiable functions is differentiable, and

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{\frac{df}{dx} \cdot g - f \cdot \frac{dg}{dx}}{g^2}$$

**PROOF:**

**EXAMPLE 5:**
Differentiate: $f\left(x\right) = \frac{x^2 - 1}{x^2 + 1}$

**EXAMPLE 6:**
Find the equation of the line that is tangent to the curve $y = \dfrac{x^2 + 3}{2x}$ at the point (1, 2). Support graphically.




**Second and Higher Order Derivatives:**
The notations we have seen are all notations for the first derivative. We can take the derivative of the first derivative, and so on...

$$y'' = \frac{dy'}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

$$y''' = \frac{dy''}{dx} = \frac{d^3y}{dx^3}$$

$$\vdots$$

$$y^{(n)} = \frac{dy^{(n-1)}}{dx} = \frac{d^ny}{dx^n}$$

**EXAMPLE 7:**
Find the first four derivatives of $y = x^3 - 5x^2 + 2$