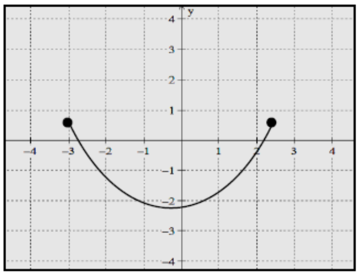
How $f'(a)$ Might Fail to Exist:

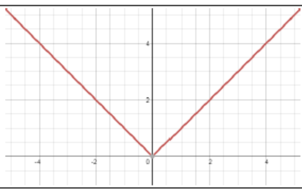
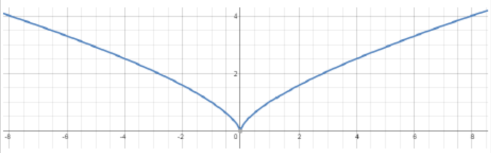
A function will not have a derivative at a point $P(a, f(a))$ where the slopes of secant lines fail to approach a limit as x approaches a.

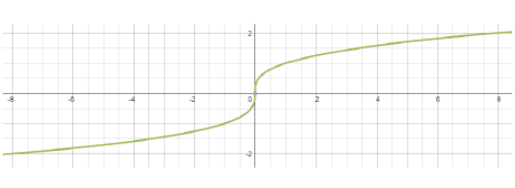
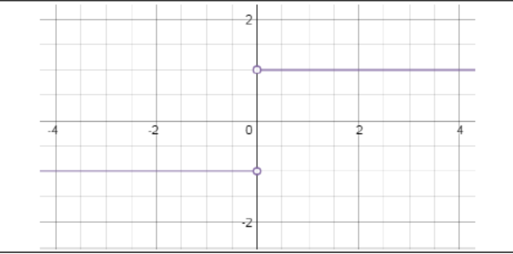
We have **five** instances where this occurs. A function whose graph is otherwise smooth will fail to have a derivative at a point where the graph has the following:

- endpoints
- a corner
- a cusp
- a vertical tangent
- a discontinuity

1. 

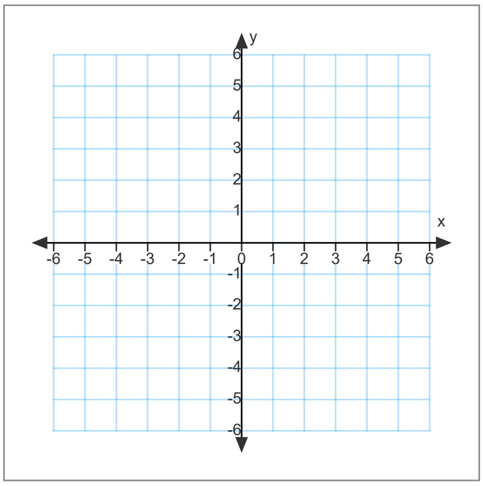
the function is not differentiable at the ***endpoints***

2.corner	
	the one-sided derivatives differ
3.cusp	
	the slopes of the secant lines approach ∞ from one side and -∞ from the other side (extreme case of a corner)

4. vertical tangent	
	the slopes of the secant lines approach either ∞ or $-\infty$ from both sides (in this example, ∞)
5. discontinuity	
	causes one or both of the one-sided derivatives to be nonexistant

<u>EXAMPLE 1:</u> Finding Where a Function is Not Differentiable

Find all points in the domain of $f(x) = |x + 4| - 1$ where f is not differentiable.



Differentiability Implies Continuity:

We have seen that a discontinuity in the graph of f would cause one or both of the one-sided derivatives to be nonexistent. In fact, continuity is an essential condition for the derivative to exist, and we will prove that.

> **THEOREM Differentiability Implies Continuity**
>
> If f has a derivative at $x = a$ then f is continuous at $x = a$.

Proof:

Intermediate Value Theorem for Derivatives

Not every function can be a derivative. A derivative must have the Intermediate Value Property, as stated in the following theorem.

> **THEOREM Intermediate Value Theorem for Derivatives**
>
> If a and b are any two points in an interval of which f is differentiable, then f' takes on every value between $f'(a)$ and $f'(b)$.

EXAMPLE 2:

Does any function have the Unit Step Function as its derivative?

$$f(x)=\begin{cases}-1, & x<0\\ 1, & x\geq 0\end{cases}$$