## CHAPTER 3: DERIVATIVES

### 3.1 Derivative of a Function

- definition of a derivative
- notation of derivatives
- relationships between graphs of $f$ and $f'$
- one-sided derivatives

Practice Problems:

P 105 # 1, 5, 13-16, 21, 22, 27, 31

### What is the derivative?

- a measure of the *steepness* of the graph of a function at a point
- the instantaneous rate of change of a function at a point
- the slope of the tangent line to the graph of the function at a point

The derivative is also, itself, a function: it varies from place to place.

**DEFINITION: Derivative**
The **derivative** of the function $f$ with respect to the variable $x$ is the function $f'$ whose value at $x$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

**EXAMPLE 1: Applying the Definition**

Differentiate $f(x) = 2x^3 - 1$

**DEFINITION: (ALTERNATE)Derivative at a Point**
The **derivative** of the function ***f*** **at the point *x = a*** is the limit

provided it exists.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

**EXAMPLE 2: Applying the Alternate Definition**

Differentiate using the alternate definition: $f(x) = \sqrt{2x+1}$

**Notation for Derivative:**

| | |
|---|---|
| $y'$ | "y prime" |
| $f'(x)$ | "f prime of x" or<br>"f of x prime" |
| $\dfrac{dy}{dx}$ | "dy by dx" or<br>"the derivative of y with respect to x" |
| $\dfrac{df}{dx}$ | "df by dx" or<br>"the derivative of f with respect to x" |
| $\dfrac{d}{dx}f(x)$ | "d by dx of f of x" or<br>"the derivative of f of x" |

**Comparing graphs of *f* and *f'*:**

When we are given the explicit formula for a function then we can find the formula for its derivative as well. Functions can also be derived graphically. We can get a good idea of what the graph of the function *f'* looks like by estimating the slopes at various points along the graph of *f*.

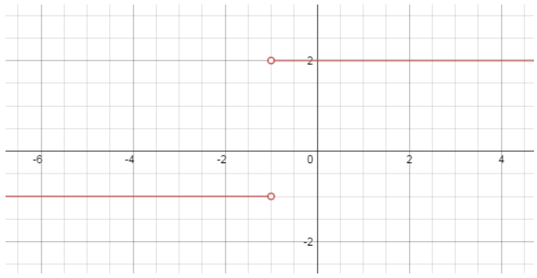
STOP **Graphing Relationships Activity!**

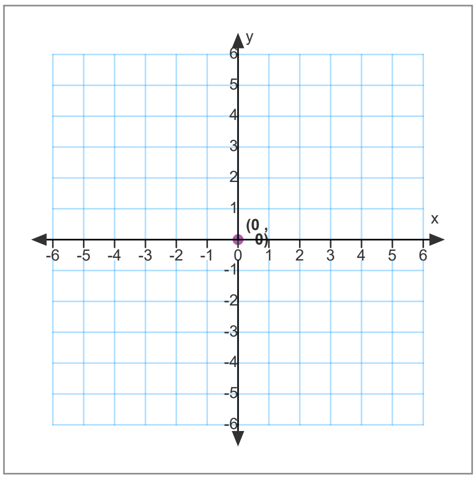
In groups of 2 or 3 you will answer the questions on the handout based on the graphs handed to you by your teacher (me).

## <u>EXAMPLE 3:</u> Graphing *f* from *f′*

Sketch the graph of a function *f* that has the following properties:

- $f(0) = 0$;
- the graph of $f'$ (the derivative of *f*) looks like this:
- *f* is continuous for all *x*




**DEFINITION: One-Sided Derivatives**

A function y = f(x) is **differentiable on a closed interval [a, b]** if it has a derivative at every interior point on the interval and if the following limits exist at the end points:

$$\lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$ the **right-hand derivative at *a***

$$\lim_{h \to 0^-} \frac{f(b+h) - f(b)}{h}$$ the **left-hand derivative at *b***

**EXAMPLE 4: One-Sided Derivatives**

Show that the following function has left-hand and right-hand derivatives at $x = 0$ but no derivative there.

$$y = \begin{cases} x^2, & x \leq 0 \\ 2x, & x > 0 \end{cases}$$